Designed Experiments In Mathematics Education Research

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In this article, I discuss the methodology of Design experiments and the prototypical characteristics of the methodology that differentiate Design experiments from other methodologies. I continue with a discussion of my experience of conducting design Experiments for my research study.

Design Experiments

The educational system can be described as "complex, nonlinear, organic, historical, and social, historical, and social. It embodies the values and problems of the society. Educational researchers use the tools of science to construct a professional language within the field of education" (Kelly, 2003, p.3) so the emerging research methodology can be both scientific and educational.

Such methodology is best described by the Design-Based research methodology that is "being situated in a real educational context" (Anderson & Shattuck, 2012).

Cobb, Confrey, diSessa, Lehrer, and Shauble (2003) elegantly describe Design experiments in Educational research, an approach to design research that has its roots in Russian teaching experiments, Piagetian psychology, and radical social constructivism (Steffe & Thompson, 2000). They highlight the complexity of the educational system by using the term *learning ecology*. Learning Ecology is "a complex, interacting system involving multiple elements of different types and levels such as the tasks that the children are asked to deal with, the kinds of discourse that are encouraged, the norms of participation that are established, the tools available in the learning environment" (p.9). They argue that in order to gain a better understanding of a complex learning ecology, we might systematically design its elements. As a result, successive patterns in students' reasoning will emerge.

In fact, *Design Experiments* address the stages of scientific processes of discovery, exploration, confirmation, and dissemination to better understand the complexity of educational systems.

Here ecological validity plays a fundamental role in charting how certain elements of the learning ecology in design experiments are actually designed to function together, in order to support target forms of learning in their "natural" or conventional settings. Ecological validity can be better demonstrated by addressing, in the research study, as many elements of the design and features in the resulting learning ecology as possible. Particular care should be

exercised though, because the more elements and collection of activities are incorporated and accounted for, the more difficult it is to comply with central ethical tenets of the research study, such as, anonymity, non-traceability and non-identifiability.

Certain crosscutting features apply to a diverse range of design experiments.

The purpose of a design-research project is to develop specific theories about both the forms of learning and the means that are designed to support them. The means can be affordances and constraints of material artefacts, teaching and learning practices, policy levers.

The second feature is the highly interventionist nature of the methodology. The design of those interventions is a key characteristic of the quality and results of the research study, so the selection and creation of the intervention is a collaborative task of both the researchers and the practitioners.

Design experiments always have two faces: prospective and reflective. On the prospective face, designs are carried out with a hypothesized learning process without neglecting of course the means which support that particular form of learning.

Careful reflection on the details of that process of design experiments provides the researcher with insights into the details, by which he can capitalize "on contingencies that arise as the design unfolds" (Cobb et al., 2003, p.10).

At the beginning, the research team or the individual researcher envisions learning in a certain setting and frames selected features of the hypotheses about how learners might develop a deeper understanding of target mathematical concepts. The initial design is typically framed by the existing literature review in the particular content domain, the researchers' personal knowledge, and pilot studies that examine students' interactions with the particular mathematical concepts and the means provided to support the emergence of those concepts. Subsequently, the design-researcher, while examining students' interactions with those means (which are under development as well), makes conjectures about how the means support effectively a particular form of learning (and vice versa) in such a way that understanding of design and the meanings that students develop are mutually constructed. In this sense, each design stage incorporates a set of conjectures about both students' mathematical abstraction and the design of means such as tools (computational or not). According to a design research methodology, these conjectures are in effect tested in the next phase of using the design product. If a conjecture is refuted, an alternative conjecture is generated and tested. "This results in an iterative design process featuring cycles of invention and revision" (Cobb et al., 2003, p.10). As the project unfolds, the design-researcher's efforts increasingly converge into an integrated coherent framework. This emergent framework, in turn, brings together principles of design for mathematical learning tools and contextualizing activities.

My approach was to adopt the approach outlined by Cobb et al. (2003) in order to develop a product (design-for-learning) but also put forward new theories of abstraction in context (theory-of-learning). Each iteration of the research was characterized by a simulation design for students to use. I conceived of each design as a conjecture.

The next section of this paper discusses the research questions of the research study.

Research Questions

The study explored the cognitive processes that were involved in connecting the two epistemological perspectives on distribution. The computer setting enabled the detailed study

of students' cognitive processes through the articulations of their meaning-making during their activity within the computer-based simulation. The study examined how the students used features of the simulation to construct meanings about the data-centric (experimental) and modelling (theoretical) perspectives on distribution and how they co-ordinated these two perspectives (Prodromou, 2008).

This intention can be brought into focus in the form of the following questions that this study intended to answer:

1) How do students account for variation in a carefully designed computer simulation?
2) How do students co-ordinate the two epistemological perspectives?

The next section of this chapter examines on the methodology of the study, examines the approach in greater detail.

Method

To elaborate the research questions, I aimed first to instantiate the conjectures into a simulation that could perturb students' thinking-in-change (Noss & Hoyles, 1996). The aim of each iterative study was to achieve articulation of emergent findings and integrate them into the model. Thus, each iteration had two phases: simulation development and simulation-use, both of which are described below.

Simulation development. The simulation development phase was marked by the search for a purposeful context that might act as a window on the students' thinking-in-change about distribution and the modifications of the emerging *BasketBall* simulation (Prodromou, 2008) during the successive iterations. In fact, the iterations articulated emergent findings and integrated them into the model. Thus, the simulation design was informed by analysis of the students' use of the simulation made available in the previous iteration. Ultimately, simulation development emerged out of this analysis.

Simulation-Use. The simulation-use phase focused on the use of the simulation by students. The activities were designed so as to support the students' meaning-making processes and provide opportunities for eliciting their intuitive reasoning about the stochastic. Specifically, analysis of students' actions and utterances during simulation-use, their interactions with the tools, the ways in which the tools were used, and the relationship between the emerging tools and the emergent expression of stochastic/statistical intuitions, constituted the data generated through the simulation-use phase of each iterated study.

Iterative Design

The research study developed through four iterations. Each iteration was roughly described in Table1. These iterations will serve to explain how the methodology itself developed in the light of outcomes from previous iterations.

This glimpse of the four iterations gives an image of the change in attention from simulation development to simulation-use.

In the beginning, the initial simulation was designed based on my initial intuitions as to how a context might act as window on the students' thinking-in-change about distribution whilst at the same time encourage focus on distribution as a central concept.

Table 1
A glimpse of the four iterations

Iteration 1	**Iteration 2**
In this first iteration, the undergraduates explored a NetLogo computer-based model. The aim was to initiate the process by making a first guess at what appropriate simulation might act as a window on students' thinking-in-change about distribution. Five undergraduate students tried out the model focusing on emergent distribution as a central concept. A major issue raised by the first iteration was that the design failed to generate purposeful student activity.	The first iteration indicated that I needed to provide students with a setting that would inspire a deterministic interpretation of behaviour and would find purpose adopting a perspective that sees behaviour captured and explained in terms of emergent distributions. I have approached the problem by setting exploration in the context of playing basketball. The process of developing the simulation made me particularly aware of the aspect of macro and micro levels. This iteration gave rise to some early ideas of how the simulation might be developed. The emphasis is both on the development of the simulation and its use.
Iteration 3	**Iteration 4**
Insights gained from iteration 2, have informed one design element that was embodied into the new simulation. This is a stage in which the simulation is sufficiently advanced and the students' interaction within it indicate support for my conjecture that it is possible to design an environment in which students' well-established meanings can be exploited to co-ordinate data-centric and modelling aspects of distribution.	The final iteration involves relatively minor modifications to the simulation which was used systematically to test my conjecture. This iteration involves much more careful and systematic analysis of how the simulation has impacted on the students' intuitions.

These initial intuitions were informed by my interpretation of the literature review together with personal knowledge of the domain and evocation of memories of personal experiences with my own students when I was teaching mathematics and statistics.

A major issue raised by the early iterations was that relatively small amounts of data generated extensive simulation development. In contrast, as the design unfolded multiple forms of data from students' activities adequately prompted minor modifications to the simulation. In fact, the minor modifications show that the iterated studies incrementally coalesce into a fairly coherent design. I observed a similar change in emphasis as noted by Pratt (1998) who noted that "The lack of change in the software was a useful heuristic signalling that the iterative process was converging towards a fairly stable design" (p. 93).

The simulation-use phase was conducted through clinical interviews. In clinical interviews, the students were asked to perform carefully designed tasks in front of me, and with prompting from myself. The dialogue that was held between the students and me centred around a task "which has been chosen to give the subject every opportunity to display

behaviour from which mental mechanisms used in thinking about the task or solving the problem can be inferred" (Hunting & Doig, 1995, p. 114).

The students were not interviewed prior to their engagement in a sequence of activities with the simulation.

All the iterations aimed to create a small-scale version of the learning ecology which could be further studied in depth and detail in the following iterations.

There was an increase in the number of students involved at each iteration, reflecting the move from exploration to more methodical analysis. The stage of the simulation-use consisted of clinical interviews around 4–5 research tasks. Each task comprised of a set of 'dynamic' activities to be shared out by a group of students and constructed at a portable computer. The activities required students to address their understandings about distribution— both their personal understandings and group understandings. The aim of the interviews was two-fold. First, the aim was to gain a general impression as to whether the age of students was ripe for conceptual change in making a causal link between features of the experimental setting and emergent distributions. Second, to inform me whether the simulation generates purposeful student activity whilst at the same time encourages focus on distribution as a central concept and support students' thinking as it moves from a micro-only perspective to one which flexibly connects a macro emergent perspective to that prior micro view.

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