Juxtaposing Modes of Inquiry

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In 2009, nationally representative samples of Year 4 and 8 students in New Zealand were asked to solve missing number problems as a part of a mathematics assessment that was video recorded. Students' numerical responses were examined using established statistical tools with an aim to explore students' conceptions of equality. As part of the same inquiry, a micro-analysis of students' verbal and non-verbal responses was conducted with a sub-sample of the video recordings. When these two methods were juxtaposed, questions emerged about the theoretical compatibility and coherence of conducting inquiry with diverse methods and multiple levels of scale.

Introduction

Juxtaposition is the act of bringing elements close together in order to compare or contrast them. In the case of my doctoral studies, disparate modes of inquiry have been chosen to further my examination of students' conceptions of equality (Anakin & Linsell, 2011). One project, currently in progress, provides the context for a methodological discussion that brings an important issue to the surface; the deficit characterisation of students' conceptions of equality.

Context

Data collected by the Educational Assessment Research Unit at the University of Otago provided an opportunity conduct inquiry using nationally representative samples of students. The Educational Assessment Research Unit implemented the National Education Monitoring Project from 1995 to 2010 to assess and report what New Zealand students know and can do at Year 4 and Year 8 of schooling. As part of the project's four year curriculum cycle, mathematics was a focal area of assessment in 2009 (Crooks, Smith, & Flockton, 2010). Of the 56 number and algebra tasks administered to students that year, one number knowledge task, reported as *link task 2*, was video recorded while it was completed by 437 Year 4 (8 and 9 year olds) and 422 Year 8 (12 and 13 year olds) students in a one-to-one interview context with an adult assessor. Students were prompted to give verbal responses and explanations as they solved three additive arithmetic missing number problems (see Figure 1). The National Education Monitoring Project team coded students' responses according to their correspondence to the coding rubric shown as part of Figure 1. A total score was obtained by adding the scores for the six of the task components coded by the team. The total scores for

link task 2 are shown in Table 1 as they were reported in the Mathematics Assessment Results 2009 (Crooks et al., 2010).

I'm going to show you some problems with numbers that are missing.

(Hand student card and point to problem 1. 5 = 3 + □)

Could the missing number be 8?

(Record student's answer.)

[Score: 1 if "no", 0 for any other response]

Explain why you say that.

[Score: 1 if "8 + 3 does not equal 5" or similar response, 0 for any other response]

(Point to problem 2. 7 + 3 = □+ 2)

What is the missing number?

(Record student's answer.)

[Score: 1 if "8", 0 for any other response]

Explain why you say that.

[Score: 1 if "8 + 2 is the same as 7 + 3" or similar response, 0 for any other response]

(Point to problem 3. 2 + □ = □+ 6)

What do you think the two missing numbers could be?

(Record student's answer.)

[Score: 1 if second missing number is 4 less than first missing number, 0 for any other response]

Could you have any other numbers?

(If student answers yes ask "*what numbers?*")

(Record student's answer.)

Explain why you say that.

[Score: 1 if "yes", 0 for any other response]

Figure 1: Assessor protocol (verbal prompts in italics, non-verbal prompts in parentheses) and coding rubric (in square brackets, not given to assessors) for *link task 2*

Table 1

Percent of students with each score for link task 2, as reported by Crooks et al. (2010, p. 40).

Score (out of 6)	Year 4 (%)	Year 8 (%)
5 or 6	6	40
3 or 4	19	25
1 or 2	38	23
0	37	12

When students' scores were interpreted as demonstrations of the concept of equality, the results became a starting point for further inquiry. To explore why such large proportions of students could be classified as demonstrating an absence or limited concept of equality, the video recordings of the three problems were examined. Two subsamples of *link task 2* student data (n = 120 at Year 4 and n = 120 at Year 8) were randomly sampled from the original data set. Instead of classifying students' verbal responses according to the criteria shown in Figure 1, students' verbal and non-verbal responses were examined as acts of communication (Sfard & Kieran, 2001; McNeill, 1992; 2005). These acts of communication were subjected to further analysis using a general inductive approach (Thomas, 2006), to reveal the specific ways in which these students were expressing their conceptions of equality. Students' verbal and non-verbal acts of communication revealed that students were not necessarily recognising a problem's features in mathematically expected ways. Instead the students referred to, and shifted focus among a variety of features including the missing number, specific combinations of the problem's elements, and the problem as a whole.

Preliminary findings suggest that students' conceptions of equality are context specific and are fragilely contingent on their knowledge of number, operations, the number system, and mathematical notation. These findings enhance our understanding of established cognitive models and add to previous accounts of how students develop the concept of equality. Falkner, Levi, & Carpenter (1999) theorised that a large proportion of the sixth-grade students in their study held an operational misconception of the equals sign instead of a correct relational one. This type of unidimensional model of student conceptions of equality has been elaborated by Rittle-Johnson, Matthews, Taylor, & McEldoon (2011). They developed a four-level construct for mathematical equivalence that theorises children move from a "rigid operational" conception to a "comparative relational" one. To the four-level model, Jones, Inglis, Gilmore, & Dowens, (2012) added the idea that substitution is a distinctive feature of the equals sign. However, all of these frameworks foreground the role of the equals sign at the expense of other mathematical features of the problem. The preliminary findings from this study suggest that in addition to the equals sign, students must successfully interpret interplay among several mathematical features of a particular problem in order to express an appropriate conception of equality.

Juxtaposing Modes of Inquiry

Since my aim is to examine how students understand the mathematical concept of equality in a productive manner, I have therefore adopted a constructivist methodology to make this concept visible for examination. Constructivist methodology has a well documented history in educational research and it is supported by ongoing cycles of psychometric instrument development, refinement, and application (Brennan, 2006). More specifically, constructivist methodology has been a dominant approach in mathematics education research and has provided insights useful to the teaching of mathematics in contemporary constructivist learning environments (Graf, 2009; De Corte, 2010).

In relation to this study, constructivist methodology supports the use of disparate methods (Crotty, 1998) and the use of cognitive representations as the unit of analysis (Hiebert & Lefevre, 1986). Constructivist methodology enables students' cognitive representations to be viewed as embedded features of the verbal and non-verbal acts of communication that occur between a student and assessor during the assessment. The phrase "conceptions of equality" has been operationalised to denote the particular, situated, and

enacted nature of students' cognitive representations as they were expressed in different ways by students when they were solving the problems in *link task 2*. Also, the term "concept" has been operationalised to denote the formalised and abstract mathematical ideas about equality that are symbolised in the problems of *link task 2*. In this way, students' conceptions of equality and the concept of equality can be compared in productive ways (Bergman, 2010; Greene, 2007; Jones et al. 2012), and in more detail than originally examined and reported by Crooks et al. (2010).

To make a detailed and productive comparison, two modes of constructivist methodology were used that operate at different levels of scale; the macro-level and micro-level. Each entails certain affordances and limitations to how the concept and conceptions of equality can be studied. At the macro-level, exploratory statistical methods reveal patterns of groups of students' responses and allow for comparison of those patterns with the concept of equality and identified conceptions of equality. In contrast, at the micro-level, exploratory inductive methods focus on the particular features of specific types of student responses to reveal new conceptions of equality because they allow for a nuanced examination of the data's features. If particular features are found to be repeating, then findings from micro-level have the potential to be scaled and analysed at the macro-level (Castro, Kellison, Boyd, & Kopak, 2010).

The concept of equality remains a challenging concept to theorise and examine, despite the rigorous application of macro-level constructivist methodology, because "it cannot lead to the identification of novel conceptions that may exist but have not already been hypothesized" (Jones et al., 2012, p. 173). Because of the possible interactions between a large number of mathematical features of particular problems, hypothesizing conceptions to account for the data is not at all straightforward. Therefore probing students' specific conceptions of equality at the micro-level is a productive way to contribute to an enhanced understanding of the patterns of student conceptions of equality at the macro-level.

As the inferences from these two modes of inquiry are juxtaposed, there is the possibility of triangulation or complementarity as well as the potential for paradoxes, contradictions, and divergences (Greene, 2007; Tashakkori & Teddlie, 2008). One issue to become apparent is how student conceptions of equality are characterised as deficits.

An Emergent Issue

The use of deficit language by mathematics education researchers is a profound issue to arise from using these two modes of inquiry relating the preliminary findings to accounts of student's conceptions of equality the reported literature. For example, students are reported to lack understanding or misapply strategies, and they are theorised to hold misconceptions (e.g., Knuth, Stephens, McNeil, & Alibali, 2006; Seo & Ginsburg, 2006). This deficit portrayal of students' conceptions equality is consistent with the way students' mathematical knowledge and skills has been researched and discussed historically. This deficit view has ensnared and put a stranglehold on the way mathematics education researchers have been able to discuss students' mathematical skills and knowledge (see Graf, 2009). Moreover, if student demonstrations of mathematics knowledge are predominantly conceptualised in relation to their correspondence to formal mathematical concepts or as conceptions framed by deficit terminology, then we miss opportunities to explore and theorise students' absence of appropriate mathematical conceptions from other points of view.

The design and findings of this study challenge the deficit characterisation of student conceptions of equality. By using growth-oriented language to describe what students already know and can do, students' absence or limited demonstrations of an appropriate mathematical conception of equality are viewed as goals for learning rather than deficits. The shift in terminology from "inadequate" cognitive representations of mathematical skills and concepts, to language that describes how students develop the ability to recognise and coordinate a number of mathematical features they encounter, is in alignment with constructivist classroom pedagogy (Istance & Dumont, 2010). Importantly, the deficit characterisation of students' conceptions of equality may not have come to the surface as an issue if different modes of inquiry had not been juxtaposed in this study.

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