Phenomenographic Investigation of Rate

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This study investigated the conceptions of the mathematical notion of rate held by 20 Victorian Year 10 as revealed by phenomenographic analysis of interviews in which computer-based simulations provided a focus for discussion. The employment of phenomenography to reveal conceptions of rate is described in detail. This analysis produced an outcome space of eight conceptions of rate based on four dimensions of variation. The efficacy of phenomenography for investigating mathematical conceptions is discussed.

Introduction

The difficulties experienced by some calculus students with the mathematical notion of rate may be a result of pre-existing conceptions (Robert & Speer, 2001). However, little is known about what these may be. Some researchers (for example Orton, 1983; Ubuz, 2001) have used quantitative methods to highlight the difficulties with some calculus students' understanding of rate. Quantitative methods give numeric measures based on statistical analysis of responses to items testing a predetermined hypothesis (Patton, 2002). Silverman (2005) suggests qualitative methods "have come up with new findings that would be difficult to establish through quantitative methods" (p.14), so since no predetermined categories conceptions of rate were available a qualitative methodology was needed to reveal the categories of conceptions of rate held by a group of Year 10 students. These categories could inform teachers of possible pre-existing alternative conceptions of rate on which to base the teaching of calculus.

In this study, phenomenographic analysis of computer-based interviews identified categories of conceptions and presented them in the form of an outcome space (see Figure 2). This methodology was deemed appropriate due its demonstrated efficacy in research in a range of educational contexts (Marton, Runesson, & Tsui, 2004). The following sections outline the methodology and then describe the application of phenomenography in this study.

Phenomenography

Phenomenography systematically seeks to describe a limited number of qualitatively different categories based on the responses of the group as a whole, providing "a description on the collective level, and in that sense, individual voices are not heard" (Marton & Booth, 1997, p.114). So, "categories of description are not intended to necessarily correspond to the perception of any particular individual … [rather they are an] aggregate of similar perceptions … [where] texts [are examined] as a group rather than singularity of individual experience … [so that] any one transcript could contribute to a number of categories" (Akerlind, Bowden, &

Green, 2005, p. 92). It is an "interpretive research approach that seeks to describe phenomena in the world as others see them" (Bruce, Buckingham, Hynd, McMahon, Roggenkamp, & Stoodley, 2004, p.147). The aim of phenomenography is to reveal categories of description delineated by the dimensions of variation which emerge from the data and hence, structure the categories into an outcome space.

Phenomenography is similar to grounded theory in that the categories or themes emerge from the analysis of the transcripts as a whole through iterative processes of iteration (Akerlind et a., 2005) rather than forcing the data to fit into a pre-determined model. Phenomenography also seeks to reveal the finite set of categories of conceptions of a phenomenon, but it is different from grounded theory because the categories are logically connected in an outcome space, usually displaying a hierarchical nature.

The process of phenomenographic analysis begins with a set of meaning statements taken from the responses in the transcripts of the interviews. The focus of the meaning statements is on the different ways the phenomenon is seen by the participants. Meaningful responses are taken from the individual transcripts and pooled, shifting attention from the individual to the meanings expressed by the group as a whole (Marton, 1988). Meaning statements are grouped to form the first tentative categories of description according to the features and characteristics (dimensions of variation) which they hold in common. The dimensions structure the description of each category. They delineate the categories one from another according to the number of dimensions or values of dimensions which are evident in each category (Cope, 2000). The dimensions facilitate an ordering of the categories into an outcome space where each category is placed in relationship to the other categories. An iterative process refines the categories and dimensions with repeated interrogation of the data.

For a phenomenographic study "the validity of the outcomes is related to the processes that are used at all stages of phenomenographic research" (Akerlind et al., 2005, p. 89), that is: the selection of the sample for maximum variation; the conduct of consistent, open interviews; and the iterative analysis which remains focused on the meanings expressed in the transcripts taken as a whole.

Method

The participants in this study were twenty Year 10 students, from five different schools, who had previously studied constant rate and some functions where rate varies. It was expected that they would provide a variety of responses consistent with the phenomenographic sampling strategy of maximising variation in perceptions of the phenomenon (Akerlind et al., 2005.

The interviews consisted of a discussion of interactive computer simulations of real-world contexts involving rate: a window partially covered by a blind, in Geometer's Sketchpad; and characters walking, in JavaMathWorld. Each simulation showed a real world context and multiple mathematical representations of the functions involved in the simulations. Phenomenographic analysis, consisting of the refinement of the categories through an iterative process involving repeated reference to the interview transcripts and videos, was then undertaken.

Results

During analysis of the data, four dimensions of variation emerged: *Focus on word 'rate'*; *Focus on variables*; *Focus on relationship between variables*; and *Focus on nature of related variables*. The dimensions formed the basis of discriminating between the categories by identifying which dimensions and which values are in focus for each category. For example, in

Category B, only awareness of rate as a numeric quantity of the dimension of variation *Focus on word 'rate'* is discerned. No other dimensions or their values are recognised.

Focus on word 'rate'
Quality
Quantity

Focus on variables
One
Two

Focus on relationship between variables
Qualitative
Quantitative constant
Quantitative Variable

Focus on nature of related variables
Distance & time
Both contexts

Figure 1. Summary of the dimensions of variation. (Herbert & Pierce, 2011)

The outcome space (Figure 2) consists of eight categories that are illustrated by examples of excerpts from student interviews and structured according to the dimensions of variation.

A	Rate is experienced as a word judging a quality.
B	Rate is experienced as a word associated with a numeric value
C	Rate is experienced as a single quantity
D	Rate is experienced as the result of a formula calculation without recognition of the relationship between quantities
E	Rate is experienced as a relationship between changes in two quantities.
F	Rate is experienced as a constant numeric relationship between changes in two quantities.
G	Rate is experienced as a numeric relationship between changes in distance and time i.e. speed.
H	Rate is experienced as a numeric relationship between changes in any two quantities.

Figure 2. "Rate Outcome Space".

Conclusion

Phenomenography was a suitable methodology for exploring conception of mathematical notions. The outcome space may provide a guide to determining the current condition of an individual student's conception of rate, which may suggest strategies to move their conception towards a more correct or complete concept of rate.

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